PROSPECTIVE SECONDARY TEACHERS’ SUBJECT MATTER
KNOWLEDGE AND PEDAGOGICAL CONTENT KNOWLEDGE OF
THE CONCEPT OF FUNCTION

By

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ABSTRACT

The current study examined two prospective secondary mathematics teachers’ subject matter knowledge and pedagogical content knowledge of the concept of functions as they completed a function questionnaire, card sorting activity, analysis of two lesson plans and their video recorded teaching, preparation and analysis of lesson plan on exponential functions, and video teaching episode of the lesson plan on exponential functions during six weeks of a teaching sequence. The task-based activities that the prospective secondary mathematics teachers were involved in during this study were analyzed to gain insight into their knowledge of the concept of functions. Even’s (1989) framework was used to examine the participants’ subject matter knowledge and pedagogical content knowledge of the concept of functions. The model of Wilson, Shulman and Richert (1987) was used to organize the tasks and investigate whether or not their pedagogical content knowledge improved as a result of doing these tasks. The study also examined the relationship between the prospective secondary mathematics teachers’ subject matter knowledge and pedagogical content knowledge of the concept of functions.

The results of the study revealed that the participants mentioned the nature of univalence property but did not know the arbitrary nature of functions. The study also revealed excessive use of the vertical line test without explaining what it meant to fail the test, why it worked or why it was or was not a function. The participants’ understanding of functions among different representations was very weak and not connected. Participants’ weak subject matter knowledge as well as weak pedagogical content knowledge prevented them from taking students misconceptions, sources of incorrect solutions, into account. The participants did gain some experience in preparing and teaching a lesson. However, they did not show a big improvement during the lesson plan analysis. In their teaching and the evaluation of their teaching, they pointed out important issues: strong and weak aspects of lesson plans, things to change in their lesson, and weakness and strength of their teaching.